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Object *a* in Numbers

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"I is an other."
— Arthur Rimbaud

Introduction

Numbers and language share a similar structure; they are both comprised of abstract objects that are used to communicate, represent, create consensus, and so on. Although numbers are used mainly for counting and measuring, and language is generally used for speaking and writing, both must follow their respective orders to make sense. If there is a crucial difference between them, it is the way they position themselves towards an unnamable or uncountable something, an object beyond their knowledge. Lacan calls this something the object *a*. In this paper I will explore the nature of the object *a*, the object-cause of desire, with reference to the nature of numbers, as Lacan does in Seminar XVII. Needless to say, language and numbers are incommensurate and thus cannot be reduced to a common denominator. If we carefully investigate the object of each structure, however, —be it a mathematical theorem or a linguistic utterance—we may reveal something common to each of them. I hope the commonalities and differences between them will help us to consider the object *a* from a new perspective.

In Seminar XVII, Lacan explores the contemporary social order and theorizes four different discursive structures: the master's, the hysteric's, the university's, and the analyst's discourse. In each, the master signifier (S_1), knowledge (S_2), the divided subject ($), and the object *a* play crucial roles in the discourse and its constitution of the subject. As Lacan writes, the object *a* "occupies the place from which the discourse is ordered, from which, if I can put it in these terms, the dominant is issued" (Lacan 2007: 43). In other words, the object *a* supports social reality in four different ways. However, the object *a* does not refer to a specific object; in fact, it should be understood as a kind of abstraction that deals with something unnamable in the experience of the social. Indeed, as Lacan writes, "it has to be said that this object is not nameable. If I try to call it surplus *jouissance*, this is only a device of nomenclature.[…] We know nothing about this object, except that it is the cause of desire" (Lacan 2007:151). Although the object *a* is the cause of desire, it is also an effect of language. Even though it is ungraspable, unnamable, and inexplicable, the object *a* is by no means something external to us. Even though it may seem contradictory, the object *a* prevents us, as an inherent impediment, from attaining satisfaction for perfect knowledge of reality. At this point, the Lacanian idea of the object *a* is related to Freud's hypothesis of the death drive, which Lacan explores as an inherent impediment preventing us from fulfilling our desire. In so doing, the drive is sustained in its unfinishable procedure. This never-ending process generates a unique pleasure in the psychic apparatus. As Slavoj Žižek writes, the course of the drive prevents "the circle of pleasure from closing" and, consequently, "irreducible displeasure" is generated. At this point, the "psychic apparatus finds a sort of perverse pleasure *in this displeasure itself,* in the neverending, repeated circulation around the unattainable, always missed object" (Žižek 1992: 56) This is how the object *a* is experienced in the psychic apparatus. As Žižek continues, "the intruder, which disturbs the harmonious circuit of the psychic apparatus run by the 'pleasure principle' is not something external to it but strictly *inherent* to it: there is something in the very immanent functioning in the psyche, notwithstanding the pressure of 'external reality,' which resists full satisfaction" (Žižek 1992: 55).

Interestingly enough, the immanent character of the object *a* is elaborated in a mathematical way in chapter XI of Seminar XVII. Lacan elucidates the nature of the object *a* with the following fractional expression: $\frac{1}{a+\frac{1}{a+\cdots}} = a$. This mathematical expression, whose denominator is repeatedly embedded in another fraction, $a + 1$, is called a continued fraction. If we personify *a*, which is expressed as $\frac{1}{a+\frac{1}{a+\cdots}} = a$, as I have just explained, $a + 1$ in the denominator, can be understood as a part of a self, called a. Then, it is not unreasonable to assume that in this formula, the object *a* is characterized by its idiosyncratic self-referentiality: *a* is constituted by referring to a part of itself. This mathematical formula indicates that the object *a* is by no means an external impediment but an internal structural apparatus. In this paper, investigating the internal nature of the object *a* from a mathematical perspective, I will illustrate how a simple mathematical formula generates a mathematical truth, an apparent totality, through an invisible Other, the law of the decimal system. However, the apparent totality of the truth in a simple mathematical formula is a fantasy supported by an implicit negativity through which ostensibly perfect knowledge emerges. The reality of the apparent totality of a simple mathematical formula is based on what Žižek calls "the radical discontinuity" between the formula and the law on the basis of which numbers in a formula can lead to a mathematical truth. The invisibility of the other does not entail a total separation between numbers and the law. Numbers and the law are neither one and the same thing, nor two different elements. They constitute a structure analogous to a Möbius strip. By aligning the Hegelian absolute subject of knowledge with the subject of science and arguing that each is a self-knowing subject, Lacan disregards the radical discontinuity between the agency of truth (numbers) and the system, which supports the reality of truth. Following Žižek's argument on the parallel between Lacan and Hegel, I will demonstrate the theoretical affinity between the Lacanian and the Hegelian subject. The structural parallel between Lacan and Hegel will be revealed in terms of the moment of inherent negation. From Žižek's perspective, one could say that the object *a* is a constitutive element of the power of Understanding, which Žižek elaborates through his argument on the Hegelian "Night of the World" as the negation by virtue of which the symbolic order emerges.

1.

First, let me begin with the following expression: 1+1. Everyone who has learned basic arithmetic will arrive at 2 as the solution of this expression: 1+1=2, "one and one make two." In this case, we assume that each number in the expression is a natural number from the set of positive integers (0, 1, 2, 3…). We follow the mathematical symbolic order in which the first symbol (+) signifies addition and the second (=) denotes the sum. When we understand the expression in this manner, "one plus one equals two," we also follow another order: a system of numeration called the decimal system. If we follow a different numerical system, such as the binary notational system (in which we have only two numbers, 0 and 1), the same expression, 1+1, one plus one, must be equal to one zero (1+1=10), because numeral concepts other than 0 and 1 do not exist in the binary system. Therefore, if the formula is in the binary system, the second 1 in the formula must interpret the + as a direction not to add itself to the first 1 to produce 2 but to add another digit, making 10 the answer to the formula. So the only possible and correct expression is 1+1=10 in the binary system. In this case, the meaning of each 1 and the expected role of + depend on the numeral system external to the formula. In this manner, a mathematical formula, 1+1, can lead to two different but correct answers; and, more importantly, the cause of this difference is invisible and detached from the formula. Before examining this issue further, it is worth analyzing the structure of the decimal system in relation to the binary system.

The decimal system is the most widely applied system of numeration having ten at its base. In the decimal system, natural numbers (0, 1, 2, 3…) are used for counting and "any number can be expressed uniquely as a sum of terms" (Peterson and Hashisaki 1963: 10). In the binary system, there are no numerical concepts other than 0 and 1. When the number 1 becomes a member of a formula whose first number is other than 0 or 1 (such as 2+1, 24+1, and so on), the 1 in each formula is a member of the decimal system because the preceding numbers (2 and 24) embody the decimal system, which is external and invisible to the formula. As agents of the decimal system, 2 and 24 in the formulas above play the role of a master who determines who can follow in the formula. Owing to the master's embodiment of the decimal system, the 1 in each formula can simply attain its identity as a member in a set in the decimal system. If the 1 could speak, it would say, "I can perceive that the previous number is neither 0 nor 1, so I immediately realize that I am a member of the decimal system because, in the binary system, neither 2 nor 24 exist." It is quite easy for the 1 in each formula to acquire its identity as a member of the decimal system. In this case, the knowledge that the 1 needs concerning its identity can be found within the formula. No sooner than it perceives the previous number does the 1 recognize its subject position within the formula as a member of the decimal system. What 2 and 24 embody is not the simple knowledge the second 1 needs; what 2 and 24 embody is the law of the decimal system to which the second 1 follows at the moment the 1 finds its subject position to follow in succession the 2 or 24 — and each declares its identity through an individual figure. So the 1 that follows does not have to ask 2 or 24 its identity, whether it is a member of the decimal or binary system, since neither of them can be agents of the binary.

But when 1 becomes a member of a formula beginning with 0 or 1, such as 0+1 or 1+1, the second 1 cannot immediately ascertain its identity or function. Unlike the 1s in the formula in the decimal system, such as 2+1 or 24+1, there is no immediate way for the second 1 to know whether the previous member is a member of the decimal or binary system. In other words, the first number in 0+1 or 1+1 can be the agency for either the decimal or the binary system. The knowledge the second 1 needs in order to decide its own identity cannot be found within the formula. If the second 1 would ask the previous number, "Are you a member of the binary or the decimal system?" the first one would reply, "It is up to you." At this moment, the second 1 recognizes the fact that it must call for something external to the formula, something it knows: "Do you know who I am? What do you want from me?"

One might say that, in order to identify its own subject position correctly within the formula, such as 1+1, the second 1 in the expression needs an Other capable of giving it an identity. In this sense, the Other can be understood as a law that interpellates numbers in a formula. The Other would say, "I know you. You are a member of the decimal system." Accepting this answer, the second 1 recognizes that it should lead to 2 in the formula — the correct answer. In this way, even within the simplest mathematical formula, such as 1+1, the identity of a number, understood as the basis on which the relation between 1's is determined, must be decided by an Other that has knowledge of each 1. What is at stake here is the fact that, when the second 1 calls upon the Other to know its identity, "who am I?" the 1 is implicitly split between 1(self) and the Other that has knowledge of it. At the price of its own identity, as in the case of the 1 in the decimal system, the 1 recognizes the fact that there is no such thing as self-knowing subject. The radical invisibility of the law supports the reality of mathematical truth. If we take numbers in a basic mathematical formula to be a self-knowing subject, we fail to take into account the status of the law, which controls not only the order of the numbers but also the identity of each number with regard to the decimal or binary system.

In order to explore the split inherent to the 1, I will discuss the concept of the unary trait. As I have demonstrated so far, in a simple mathematical formula, such as 1+1, the identity of the first 1 is determined intersubjectively in relation to the second 1 by the Other. In the decimal system, where the solution is not "1+1=10" but "1+1=2," the first 1 in the formula must be identified as a member of the decimal system by the second 1. In this sense, the very interrelation — or, rather, intersubjective relation — between the 1s represents what Lacan calls the unary trait, where both 1s are able to produce a correct expression together. In fact, with respect to the unary trait, Lacan states, "we only have to place this unary trait in the company of another trait, S_2 after S_1" (Lacan 2007: 51). Using Lacan's language, if we place the second 1 "in the company of another trait," say, the first 1, the latter can play the crucial role of S_1. Only in this inter-number moment can the identity of each I be defined. The second 1 must identify itself as the second number that must properly recognize the identity of the previous number to add itself appropriately to the first 1, whereby the solution, 2, can be produced. Thus it is noteworthy that the second 1 is the subject of knowledge that knows that the first number interactively embodies the order of the following numbers in the formula. If the second 1 identifies itself as a member of the binary system, it leads to 10 as the mathematical truth. Even though it is given prior to the second 1, the first 1 by no means acts unilaterally. The role of the first 1 is determined retroactively when the second 1 is added to the formula.

This intersubjective relation between the numbers in the formula is exactly the same as the structure of what Lacan calls the discourse of the master, in which the master signifier can be the master only when the following signifier is added with knowledge, which is provided by an Other external and invisible to the formula/discourse. In this sense, the relation between the numbers in a mathematical formula is similar to the relation between signifiers in a discourse: "the signifier becomes articulated, therefore, by representing a subject for another signifier" (Lacan 2007: 48). A number or signifier cannot be the transcendental or absolute master. Each master in a symbolic formula, be it mathematical or linguistic, is always already governed or rather castrated by knowledge to play the role of the master in the formula. In order to make sense as numerical expressions, both formulas, 1+1=2 and 1+1=10, require an Other to provide knowledge; the Other is not only unrelated to the numbers but also invisible to them, despite the control it exerts. In short, there is nothing in common between the numbers in the formula and external knowledge. This disjunction between the formulas and the Other's knowledge is structurally similar to the split between the subject of the signifier and the subject of knowledge.

In the following section, I will examine this split in terms of the contrast between the Lacanian and Hegelian subjects. Lacan argues that while the former is the subject of a split, the latter is the subject of imaginary totality. In this respect, Lacan makes a claim about the homogeneity between the Hegelian subject and the subject of science: both of which disavow the split between the subject and knowledge, which gives an identity to the subject from the outside. *Contra* Lacan, however, Žižek argues convincingly that there is an affinity between Lacanian psychoanalysis and Hegelian philosophy. Following Žižek's argument, I will demonstrate how both the Lacanian and Hegelian subjects experience the radical discontinuity between the organic immediacy of 'life' and the symbolic universe through the negation of the Real, which is covered by the object *a*.

2.

When translated into English, both *connaissance* and *savoir* are knowledge, but it is crucial to note that Lacan identifies two different kinds of knowledge: imaginary and symbolic knowledge. Imaginary knowledge (*connaissance*) is the knowledge of the ego; it is self-knowledge that is based on a fantasy of the absolute unity of the self. By contrast, symbolic knowledge (*savoir*) is knowledge of the subject, which follows the symbolic order. Symbolic knowledge resides between the subject and the symbolic. As I will demonstrate in this section, this distinction is significant when considering the differences and commonality between the Lacanian subject and a philosophical subject such as the Hegelian or Cartesian subject, particularly with respect to knowledge and jouissance.

As Lacan argues, "whereas knowledge is a means of jouissance, work is something else" (Lacan 2007: 79). My suggestion is that what numbers do in a mathematical formula can be considered in terms of what Lacan calls "work." As I have already demonstrated, to "work" appropriately in a formula, each number must obey the Other with the knowledge that indicates to which system a number belongs. Here, we can also understand that, "Even if work is accomplished by those who have knowledge, what it produces can certainly be truth, it is never knowledge—no work has ever produced knowledge" (Lacan 2007: 79). This means that, even though each number in a given formula might work properly and produce a mathematical truth—for instance, the "truth" of 1+1 is 2 in the decimal system—such a mathematical truth is by no means knowledge. In simple terms, although produced by knowledge and working according to knowledge, what numbers produce is not knowledge but truth. The numbers in the formula are subjects of symbolic knowledge and agents of truth.

In this light, the numbers in the formula 1+1 cannot have a means of addressing jouissance in their own right. Or, rather, it should be said that they work as numbers in the formula because they have renounced their ways of reaching out to the jouissance. This means that numbers in the formula are castrated by the knowledge given by the Other that is invisible and disconnected from them. If we try to understand this relationship between numbers and the Other anthropomorphically, as Lacan does in Seminar XVII, it might be said that numbers are brothers waiting to be interpellated by the mythical Father so that they can become members of a mathematical formula. By virtue of the Other's interpellation, numbers "discover that they are brothers" (Lacan 2007: 114) in the sense that the numerical difference between them does not matter when it comes to their "work" in a given formula. In fact, as Lacan states, "All signifiers are in some sense equivalent, if we just play on the difference of each from all the others, through not being the other signifiers. But it's for the same reason that each is able to come to the position of master signifier, precisely because its potential function is to represent the subject for another signifier" (Lacan 2007: 89).

It should be clear by now that knowledge external to a mathematical formula plays a crucial role in its structure, undermining its ostensible totality in the form of mathematical truth, such as 1+1=2 / 1+1=10. When we think 1+1=2 is true, we must take into account the fact that our acceptance of this truth is based on the knowledge of the decimal system, which is invisible in the formula. We follow this invisible knowledge even though we cannot clearly identify the existence of the decimal system in the formula. The decimal system itself is enacted with our acceptance that 1+1=2. In other words, our acknowledgement itself enacts the knowledge. When we accept that the formula is right, we become an agent of the knowledge of the decimal system. In this sense, the law of the decimal system castrates both the numbers and us. What should be noticed here is that, even though it may be governed by the knowledge of an invisible law, a formula seems to produce truth by itself. In other words, the mathematical formula seems to produce mathematical truth, and an apparent totality ostensibly takes place within the formula (1+1=2 or 1+1=10). One could say that, even though the mathematical formula is controlled or rather castrated by the knowledge of the Other, its invisibility covers the radical discontinuity between the formula and invisible knowledge. For this reason, the numbers in the formula can be taken erroneously as knowing subjects. Lacan's criticism of the Hegelian knowing subject—"all the knowledge is known from the outset" (Lacan 2007: 89)—should be considered in this light. What Lacan presupposes is that the Hegelian subject ignores the fact that it is always already castrated by the Other, its "mythical support" (Lacan 2007: 90), to be an agent of absolute knowledge. However, following Žižek's argument, I would like to claim that Lacan's criticism of absolute knowledge disregards the moment of the negation, the "Night of the World," which has, as Žižek points out, a structural affinity with the Real. Contrary to Lacan's argument on Hegel, Žižek finds an affinity between Lacan and Hegel by focusing on Hegel's concept of the Night of the World as the negation by which the universe of knowledge emerges. Thus negation itself is a part of the subject. In this light, the symbolic universe of absolute knowledge is inherently structured by the split between the agent of absolute knowledge and the negation of the Night of the World. At this point, as Žižek argues, Hegel and Lacan share the same constitutive presupposition: the Hegelian and Lacanian subjects pass through the zero point of the Real, which must be negated to be in the symbolic:

> One of the lessons of psychoanalysis—and at the same time the point at which Lacan rejoins Hegel—is the radical discontinuity between the organic immediacy of "life" and the symbolic universe: the "symbolization of reality" implies the passage through the zero point of the "night of the world." What we forgot, when we pursue our daily life, is that our human universe is nothing but an embodiment of the radically inhuman "abstract negativity," of the abyss we experience when we face the "night of the world." (Žižek 1992: 53)

What Žižek emphasizes here is that the symbolic universe emerges from the abyssal experience through which the radical discontinuity between the Real and the symbolic universe is generated. The point is that the world of knowledge and the abyss of the Night of the World are neither one and the same nor two separate things. They are inseparable yet different elements of subject. The negation of the Night of the World is a constitutive element of the symbolic universe, the invisible moment of negation.

When Lacan criticizes mathematics by saying "*A* represents itself, without any need for a mythical discourse to give it relation," he recognizes a homogeneous structure between the mathematical formula and the Hegelian subject that "asserts himself as knowing himself" (Lacan 2007: 89). I have argued thus far, however, that once a number is posited in a numerical signifying chain, such as 1+1, no number can be a knowing subject. No number can be a self-reflective, self-confirming, autonomous knowledge on its own account, because numbers in a formula require the Other's knowledge, which is invisible in the formula, in order to work appropriately. Thus all the numbers in a mathematical formula are what Lacan calls subjects of *savoir*, which are always already castrated by the Other's knowledge (the decimal or binary system). In this sense, although Lacan asserts that "the master's knowledge is produced as knowledge that is entirely autonomous with respect to mythical knowledge, and this is what we call science" (Lacan 2007: 90), numbers in a formula are necessarily heteronymous split subjects. More precisely, although they play a key role in scientific discourse, numbers are by no means self-knowing, autonomous subjects of knowledge. If a mathematical formulation seems to produce a truth without an external Other, as Lacan claims, it is only because the Other as the law—such as the decimal or binary system—is invisible in the formula. Put differently, the apparent truth of a mathematical formula is brought about through the negation of the invisible Other of the law, which supports the order of the formula. In this light, the numbers in a formula are by no means self-knowing, autonomous subjects.

Lacan's argument concerning the difference between the subject of *savoir* and the subject of *connaissance*, the self-knowing subject, should be examined with respect to his argument about the Cartesian subject. The Cartesian subject can be understood as a self-knowing subject insofar as it has no negation of the Real in its inherent structure. The Cartesian theorem, "I am thinking therefore I am" is, in other words, blind to the fact that it requires the Other in order to make sense. This Other is external to the *cogito*, but it is nevertheless constitutive of it. In short, the Cartesian subject unconsciously utters "I" without knowing about its reliance on the existence of the Other. When Lacan elaborates Cartesian subject in the following mathematical formula.

$$\frac{1}{1+1} = ?\ \frac{\text{I am one}}{\text{I am thinking} = \text{therefore I am one}} \quad (154)$$

Lacan argues that the structure of the Cartesian "I" is paradoxically supported by the "I's" own knowledge; this knowledge is the means by which "I am" becomes the whole "I," the "I am one." Instead of referring to the Other that governs the structure of the "I am one," "I am thinking," or “therefore I am”, Descartes relies self-reflectively on a part of the theorem, the “I am.”

Lacan argues that the Cartesian formula “I am thinking therefore I am” is fatally blind to the fact that this theorem is, from the outset, constituted by the concept of the “I am,” the symbolic order. It means that the Cartesian theorem is simply based on the fantasy of a self-knowing subject that must be blind to both the moment of the unary trait—or, rather, the intersubjectivity that makes it possible to utter “I am” without knowing it—and the being of the Other that interpellates members of the formula. Indeed, as Lacan says, “the effect of division is already marked by an ‘I am’ which elides the ‘I am marked by the one’. [. . .] It is as a function of this initial position of ‘I am’ that the ‘I am thinking’ can be even so much as written” (Lacan 2007: 155).

Lacan states that, in the Cartesian expression, “‘Therefore I am’ is a thought. It supports itself infinitely better by carrying its characteristic of knowledge, which does not go beyond the ‘I am marked by the one,’ by the singular, unique…what? —by this effect which is ‘I am thinking’” (Lacan 2007: 155). The point is that the Cartesian formula does not *go beyond*, nor does it take the unary trait into account, which “comes from the Other in order to build a singular identity over its lack of being” (Verhaeghe 2007: 32). Like mathematical expressions, which need to refer to external numeral systems to make sense, language requires symbolic structures in order to produce meaning. However, the Cartesian formula does not recognize its reliance on the external system that makes it possible to utter “I am” as a reasonable statement. So it could be said that the Cartesian formula is blind to the fact that, in order to qualify its own theorem, it relies, from the outset, on a part of the theorem itself seeking qualification. From the Lacanian perspective, the Cartesian theorem fails to take into account the cause of desire—the object *a*—because it neglects the external order that structures the theorem “I am thinking therefore I am.”

In Seminar XVII, Lacan juxtaposes the object *a* with jouissance. As Paul Verhaeghe notes, “in Seminar XVII [Lacan] develops this connection between the unary trait and the Other through the concept of jouissance. The Other marks the invasion of the enjoying body through the unary trait” (Verhaeghe 2007: 32). Because the Cartesian “I” seems to suppose that the Cartesian I is not mediated by the Other but supported by its own knowledge, the “I am” could be thought of as a self-knowing subject. In Lacan’s view, the Cartesian theorem fails to recognize that it must repeat itself to form an “I.” Concerning this philosophical subject, Mladen Dolar states that “the subject that cannot know itself, that cannot self-reflectively grasp itself, the nonphilosophical consciousness that wants to grasp the truth with its knowledge, but that perpetually fails to do so, becomes the protagonist of philosophy” (Dolar 2007: 146). The Cartesian “I” is, in fact, what Dolar calls the protagonist of philosophy that cannot understand the fact that it fails to know itself and cannot self-reflectively grasp itself. Theoretically speaking, in the light of Žižek’s argument, the Cartesian subject does not experience the Hegelian Night of the World on account of which negation converts being and makes it possible for the symbolic universe to emerge.

To elucidate the nature of the object *a* in a more explicit way as a mathematical concept, Lacan refers to the structure of the golden number. The golden number appears retroactively when the Fibonacci series is calculated long enough. As he says, “you will thus find the numbers the sequence of which constitutes the Fibonacci series, 1, 2, 3, 5, 8,… each being the sum of the two preceding numbers. [. . .] This relation of two terms, we can write for instance as $u_{(n+1)} = u_{(n-1)} + u_n$” The result of the division u_{n+1} / u_n will be equal, if the series is continued long enough, to the effectively ideal proportion that is called the proportional mean, or again, the golden number.” (Lacan 2007: 156). Then he associates the structure of the golden number with “an image of what affect is,” saying that “if we now take this proportion as an image of what affect is, insofar as there is repetition of this ‘I am one’ on the next line, this retroactively results in what causes it—affect” (Lacan 2007: 156). As Lacan states:

> It is self-evident that repetition of the formula cannot be the infinite repetition of the “I am thinking” within the “I am thinking,” which is the mistake the phenomenologists never fail to make, but only the following: “I am thinking,” were it to be done, is only able to be replaced by “I am, ‘I am thinking, therefore I am.’” I am he who is thinking, “Therefore I am,” and so on indefinitely. You will observe that the small *a* always gets farther and farther away in a series that reproduces exactly the same order of 1s, such as they are here deployed on the right, with the sole difference that the final term will be a small *a*. (Lacan 2007:157)

What is at stake here is that, if we try to denote the nature of the “I” in terms of the Cartesian theorem, “I” cannot stop saying “I am thinking therefore I am.” This means that a part of the theorem should always be repeated to represent “I.” The cause of this repetition of the “I” in self-representation is ignored in the Cartesian theorem. As his reference to the golden ratio suggests, Lacan argues that the structure of the “I” is similar to that of irrational numbers. Generally speaking, such numbers are expressed as signs, $\sqrt{2}$, π, or ψ, which can be expressed as continued fractions, as is evident in the formula of $\sqrt{2}$, I have explained above. The formula Lacan adopts to explain the object *a*, $\frac{\frac{1}{u+1}}{u+1} = u$, has a similar structure to the continued fraction formula he uses to denote the Cartesian “I” insofar as both repeat a part of themselves in each denominator to represent themself. In contrast to Descartes’ conception of the subject, in which the “I” attempts to stop repeating itself, Lacan demonstrates how the subject requires infinite repetition. In the following section I will consider the nature of the object *a* and its role in this infinite repetition.

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\ddots}}}}.$$

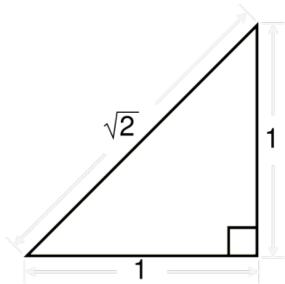
3.

Broadly speaking, numbers can be divided into two categories: real and imaginary. The former can be easily understood as points that exist theoretically somewhere on an infinite number line. The latter are numbers whose square root is a real number less than zero. What is essential here is real numbers. Real numbers can be divided into two categories: rational and irrational. Rational numbers can be expressed as a ratio of two integers, Irrational numbers, by contrast, cannot be expressed in this way. In contemporary mathematics, irrational numbers are represented with the help of Greek letters π, φ, or a particular sign like √. I would like to investigate the difference between rational and irrational numbers in order to explore the reason why Lacan refers to an irrational number to explain the object *a*.

The most familiar rational numbers are natural or whole numbers, which are used in the decimal system. Natural numbers can be used for counting, beginning with zero, such as 0, 1, 2, 3, and so on. Natural numbers can be expressed as a ratio of two integers; $3 = \frac{3}{1}$ or $7 = \frac{49}{7}$. In this way, all natural numbers can be shown in the form of a fractional expression of two integers, so it can be said that every natural number is a rational number. Theoretically, we can enumerate natural numbers infinitely.

In addition to natural numbers, there is another kind of rational number called a repeating decimal such as $\frac{1}{7} = 0.142857142857142857$ … …. When we try to show $\frac{1}{7}$ as a decimal, from the first digit after the decimal point to the sixth place, the finite sequence of digits (142857) will repeat itself, potentially to infinity. When we express this number in the form of a fraction, it seems to be a natural number, yet once we express it as a decimal fraction, a sequence of numbers, a part of the number itself begins to repeat itself beyond control. It will potentially repeat infinitely. To remain a rational number, such decimals should be expressed as a fraction. If the numbers in the sequence could speak, they would say "we must renounce our desire to repeat ourselves to (apparently) remain rational." With the help of a fraction formula, a repeating decimal can remain a rational number. In other words, to remain rational, their desire to repeat themselves must be abandoned. In this case, the difference between rational and irrational hinges on variant modes of mathematical representation. In fact, numbers unable to be expressed as fractions are irrational numbers. Some of the most famous irrational numbers are , π, and ψ. π (pi) represents the ratio of the circumference of a circle to its diameter, and ψ (phi) represents the golden ratio. To consider the relation between the nature of a number and the object *a*, let me begin by examining $\sqrt{2}$.

$\sqrt{2}$ is most often encountered in its relation to the Pythagorean Theorem, $a^2 + b^2 = c^2$. According to the Pythagorean Theorem, the square of the hypotenuse of a right-angled triangle is equal to the sum of the square of the other two sides. For example, a right-angled triangle with sides 3, 4, and 5 can be represented as $3^2+4^2 = 9+16=25=5^2$. In this case, the theorem works beautifully within the domain of rational numbers. In other words, the theorem produces mathematical truth in a Symbolic containing rational numbers. But with a right-angled isosceles triangle with two 1 unit long sides, we cannot express the length of the hypotenuse with rational numbers, since there is no rational number whose square is equal to 1. In this case, the length of the hypotenuse is incommunicable or unnamable within the domain of rational numbers. However, if we look at an isosceles triangle with leg lengths of one, we see that it is there as a discrete quantity of distance.



It genuinely exists between two straight lines making a right angle. Yet Pythagoras has no way to name it in the domain of rational numbers, the only means by which he could express mathematical truth at that time. At the moment he encounters the hypotenuse, what Pythagoras confronts is the Real in the existing Symbolic. Encountering something unnamable, he may mutter to himself (as Wittgenstein did), “whereof I cannot speak, thereof I must be silent.” But instead of keeping silent, Pythagoras acknowledges it as something outside the domain of rational numbers, something “lying beyond the infinite sequence of rationals” (Moore 1990: 22). Without a system of notation (such as $\sqrt{2}$, π, or ψ) that clearly and definitely expresses irrational numbers, irrational numbers cannot exist as numbers per se but only as a concept such as “a length of an oblique side of a right angled triangle whose straight lines are one unit long respectively.” But a concept is not commensurate with the order of natural numbers. Concepts and numbers are incommensurable with each other, so theoretically they cannot coexist simultaneously in the same dimension. In this way, a short diagonal line in a triangle fatally breaks the harmony of the world of the rational number. The imaginary totality of the Pythagorean perspective collapses at this moment. When multiplied by itself, $\sqrt{2}\times\sqrt{2}$, it turns into 2, a natural number. But without auto-multiplying, without self-repeating, it remains something unnamable in the order of rational numbers. Pythagoras happens to encounter something unnamable as an unexpected effect of his knowledge. This encounter, which undermines the fantasy of perfect knowledge, is, as Žižek emphasizes, the eruption of the Night of the World through which the symbolic universe emerges. In this way, $\sqrt{2}$ fatally demolishes Pythagorean totality. This means that the Pythagoras’ theorem, which was believed to contain the knowledge necessary to produce mathematical truth, confronts the limit of its knowledge in the specter of the irrational number. As Moore suggests, the Greek word, “‘*peras*’ is usually translated as ‘limited’ or ‘bound.’ ‘*To apeiron*’ refers to that which has no *peras*, the unlimited or unbounded: the infinite” (Moore 1990: 17). In this sense, confronting something *apeiron*, Pythagorean confronts something beyond his knowledge. This is the object *a*.

In contemporary mathematics, without using the square root sign, which was invented in the 16th century, long after Pythagoras, the numerical value now as expressed as $\sqrt{2}$ has no way to be shown concisely. Indeed, when we try to indicate this value without the symbol of the square root, we find we must resort to the law of speech and, therefore, abandon the realm of numerals. The only means we have to demonstrate it within mathematical notation are *a priori* incomplete: either in an infinite decimal, such as 1.41421356…, or with the aperiodic continued fraction whose sequence, a part of itself, will repeat infinitely in theory. In either case, we cannot stop calculating if we want to achieve the truth of $\sqrt{2}$. Something unnamable commands us to “keep calculating.” Theoretically, we can say that the truth of $\sqrt{2}$. potentially exists somewhere on an infinite number line between 1 and 1.5, or somewhere around there. But what is at stake here is the fact that no one can identify its exact position; all we can do is approximate it. Only when represented by the symbol of the square root does it seem to occupy a definite position within the symbolic universe of numbers. However, if it could speak it would say, “I am closely approximate to something you want to represent by using your arbitrary symbol, $\sqrt{2}$ but I cannot completely identify with $\sqrt{2}$. If you want to know my true character, keep calculating. Only while you are calculating can you enjoy the possibility of my truth." The point is that this imaginary voice is exactly the voice of the Real, the voice from the abyssal experience of the Night of the World. Like the voice of the monster of Frankenstein, who is not supposed to speak, the voice of the Real should not be heard, should be totally negated in order to sustain the symbolic order. The negation is inevitably preserved in order to maintain the symbolic universe. As Žižek claims, the negation is constitutive of the symbolic universe: "[T]he symbolic order, the universe of the Word, emerges only against the background of the experience of this abyss, as is demonstrated by Hegel" (Žižek 1992: 50). At this time, as Žižek argues, what we experience is the power of Understanding with which we can *tarry with the negative* in order to stay within the symbolic universe.

In the same way, π might say "I definitely represent the ratio of the circumference of a circle to its diameter" and ψ might say, "I am the golden ratio," but these statements are not true in a strict sense. The ratio of the circumference of a circle to its diameter is approximately equal to 3.14159…. And it can also be represented in an aperiodic continued fraction. Only when a device of nomenclature (π) helps us can the ratio of a circle be described as something definite. Similarly, ψ represents the golden ratio in the mathematical symbolic. But, in a strict sense, the golden ratio is denoted as, $\frac{1+\sqrt{5}}{2} \approx 1.6180339887$ ... or in an aperiodic continued fraction as $\sqrt{2}$.

How then should we understand the roles of the signifiers $\sqrt{2}$, π, and ψ, within the numerical Symbolic? In this system of notation, these irrational numbers can be represented through a definite sign. Although they are adopted to represent irrational numbers, they by no means express the true character of what these symbols appear to represent. Indeed, as Timothy Gowers, a recipient of the Fields Medal for mathematics states, "[m]ost people think of mathematics as a very clean, exact subject. […] Those who continue with mathematics at university level, and particularly those who do research in the subject, soon discover that nothing could be further from the truth" (Gowers 2002: 112). Thus we understand that these symbols can only indicate a way to approximate the truth of what they represent. When Lacan mentions that "the space in which the creations of science are deployed can only be qualified [. . .] as the *in-substance*, as the *a-thing*, *l'achose* with an apostrophe—a fact that entirely changes the meaning of our materialism" (Lacan 2007: 159), he may be thinking about a mathematical amnesia in which subjects completely forget that the symbolic order can be constructed only by abandoning the desire to know the truth.

It should be clear by this point that signifiers (such as $\sqrt{2}$, π, or ψ) can represent rational numbers as compensation for abandoning the desire to calculate the true numerical position of each irrational number. Only within such a process of calculation can we enjoy the possibility of knowing the truth. It follows that these symbols represent the loss of such a possibility, because the symbols, $\sqrt{2}$, π, and ψ, are signifiers that appear at the cost of infinite calculation—or, better, these signifiers represent the renunciation or exclusion of jouissance. By means of these symbols, we can temporarily take into account something unnamable, something beyond our knowledge. Therefore, what is at stake here is the chasm between representation and truth. This chasm is the cause behind our desire to know the truth. It is what Lacan calls the object *a*. Although we can infinitely approximate the truth of rational numbers, we can never know the exact value of $\sqrt{2}$, π, or ψ. Even if we could calculate to infinity, something is always left that repeats itself. Like the signifying chain, in the calculating chain of irrational numbers, the object *a* will be generated repetitively as the cause of desire. I have suggested here that the mathematically invented Symbolic provides us with an imaginary totality that can distract us from our desire to know the truth, from the insatiable desire to see the emergence of the impossible, that is, the unnamable.

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